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Special relativity with synchrony parameters

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Abstract - We show that the transformation equations for the space-time coordinates of the same event, derived on the basis of the constancy of the round trip speed of light, could be derived by performing the Lorentz-Einstein transformations of the event generated by a subluminal one way signal when it arrives at the location of the clock to be synchronized with the clock located at the origin of the inertial reference frame. Considering that it propagates with speed c_f=c/n (n>1), n dependent "general transformation equations are derived. Particular values of n are considered, leading to absolute simultaneity.

Keywords: special relativity, inertial reference frame

1. Introduction

The standard formulation of the postulates on which Einstein's special relativity is based states: [1]
(α) Relativity principle: All physical laws are the same in any inertial reference frame (IRF). No reference frame is "privileged" i.e. distinguishable the other (IRF)'s by means of "internal" empirical evidences.
(β_1) Invariance of the velocity of light: The velocity of light in empty space is the same in all (IRF). Its value is given by the universal constant c. It is considered [2] that (β_1) cannot be empirically tested and so they consider that (β_1) should be stated as: (β_2) The velocity of light is a universal constant c in any (IRF) along any closed path.

The two propositions (β_1) and (β_2) lead to different synchronization procedures. Both are performed in a given (IRF), say I, involving the clocks $K_0(0)$ and K(x), the first located at the origin O, the second at a point M(x) located on the OX axis. Figure 1a illustrates the synchronization of the two clocks based on (β_1), proposed by Einstein, on a classical space-time diagram.. When clock $K_0(0)$ reads t_e a source of light S(0) located at the origin O emits a light signal in the positive direction of the OX axis. It arrives at the point M(x) when the clock K(x) located there reads t_E=x/c. Being reflected back without delay it returns to the origin O when clock $K_0(0)$ reads t_r. Relativists say that $K_0(0)$ represents the wrist watch of an observer $R_0(0)$ located at the origin O. We underline that t_E is a reckoned time, the times t_e and t_r being displayed by the wrist watch mentioned above. The times t_e; t_E and t_r being related by

$$t_E = t_e + \frac{x}{c} \tag{1}$$

$$t_E = t_r - \frac{x}{c} \tag{2}$$

resulting

$$t_E = \frac{1}{2}\left(t_e + t_r\right). \tag{3}$$

The conclusion is that R_0 can assign to the event associated with the arrival of the synchronizing signal to point M(x) a time coordinate t_E knowing the corresponding readings of his wrist watch.

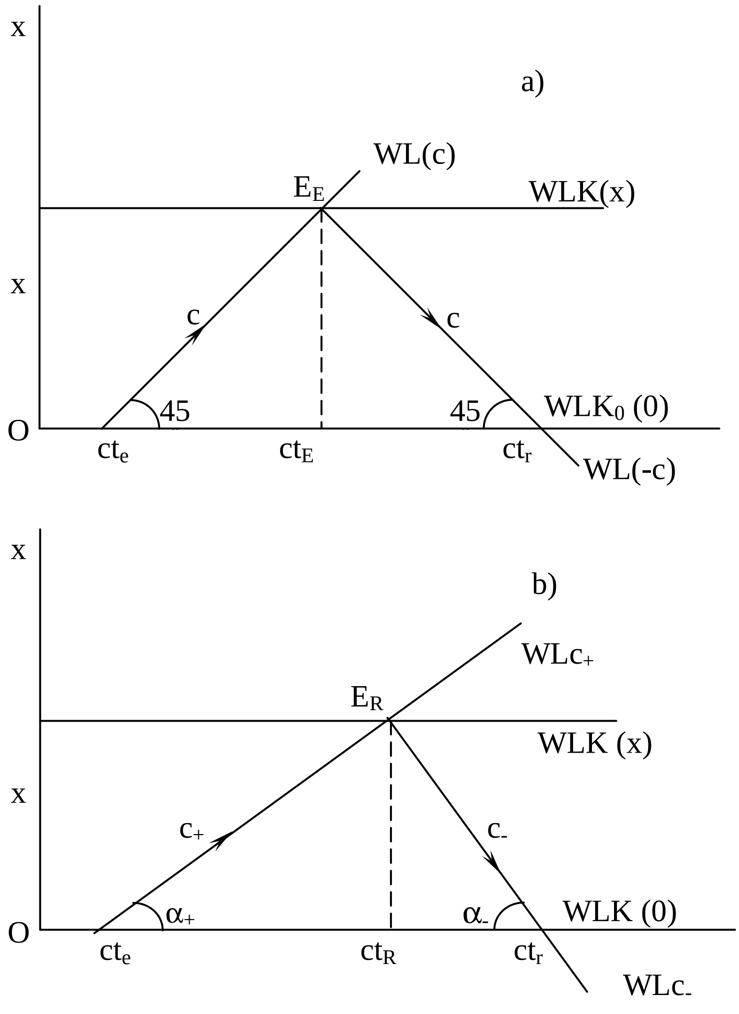

Figure 1. (a) Illustrating the synchronization of two distant clocks, following Einstein's procedure (β_1) on a classical space-time diagram. (b) Illustrating the synchronization of two distant clocks following Reichenbach's procedure on a classical space-time diagram.

Figure 1.b illustrates the synchronization of the same two clocks based on sentence (β_2). A source located at the origin O emits a signal that propagates at a subluminal speed $c_+<c$ when the clock located there reads t_e. The emitted signal arrives at the location of clock M(x) when the wrist watch of observer R_0 reads $t_R = x/c_+$ and returns back to the origin O propagating with speed $c_->c$ when the clock located there

reads t_r. The clock readings t_e and t_r are the same as in the case of the (β_1) synchronization. The geometry of the classical space-time diagram tells us that t_e; t_R and t_r are related by

$$t_R = t_e + \frac{x}{c_+} \tag{4}$$

$$t_R = t_r - \frac{x}{c_-} \tag{5}$$

Combining (1) and (4) the result is

$$t_E = t_R + \frac{x}{c}\left(1 - \frac{c}{c_+}\right) \tag{6}$$

whereas combining (2) and (5) the result is

$$t_E = t_R - \frac{x}{c}\left(1 - \frac{c}{c_-}\right). \tag{7}$$

Consistency requires that

$$1 - \frac{c}{c_+} = -\left(1 - \frac{c}{c_-}\right) \tag{8}$$

or

$$\frac{2}{c} = \frac{1}{c_+} + \frac{1}{c_-} \tag{9}$$

c in (9) representing the two way (round trip) speed of light. Equation (9) shows that the one way speed of light in special relativity theory and the round trip speed of light are equivalent.

2. Continuing with Einstein's philosophy

Einstein considering equation (6) will say that it is the result of a scenario followed from the I inertial reference frame, that involves the clock $K_0(0)$ located at the origin O and the clocks $K_1(x)$ and $K_2(x)$ located at the same point of the OX axis. Clock $K_1(x)$ is synchronized to $K_0(0)$ using an one way signal that propagates with speed c, whereas clock $K_2(x)$ is synchronized to $K_0(0)$ using a subluminal signal propagating with the speed $c_+<c$. In order to simplify the notations we consider that $c_+=c/n$ where $n>1$ with which (6) becomes

$$t_E = t_R + \frac{x}{c}(1-n). \tag{10}$$

In accordance with (α) (10) reads in I'

$$t'_E = t'_R + \frac{x}{c}(1-n) \tag{11}$$

Following in his philosophy Einstein will consider that the Lorentz-Einstein transformations

$$x = \gamma\left(x' + Vt_E\right) \tag{12}$$

$$t_E = \gamma\left(x' + \frac{V}{c^2}t'_E\right) \tag{13}$$

$$x' = \gamma\left(1 - Vt_E\right) \tag{14}$$

$$t'_E = \gamma\left(t_E - \frac{V}{c^2}x\right) \tag{15}$$

hold exactly only in the case when the times t_E and t'_E are displayed by standard synchronized clocks, V representing the constant speed with which I' moves relative to I in the positive direction of the permanently overlapped OX(O'X') axes. They should also hold exactly when we replace in them t_E and t'_E with (10) or (11) respectively Doing so a transfer takes place from the times displayed by clocks (β_1) synchronized, to times displayed by clocks (β_2) synchronized using subluminal signals.

Consider that in I' frame the clocks are synchronized using the subluminal signal, that generates, arriving at the location of clocks $K'_1(x')$ and $K'_2(x')$, the event

$$E' = \left[x', t'_E = t'_R + \frac{x'}{c}(1-n')\right]. \tag{15a}$$

Performing the Lorentz-Einstein transformations of the space time coordinates which define event (15a) the result is

$$x = \gamma\left\{\left[1 + \frac{V}{c}(1-n')\right]x' + Vt'_n\right\} \tag{16}$$

$$t_E = \gamma\left[t'_R + \frac{x'}{c}\left(1 + \frac{V}{c} - n'\right)\right]. \tag{17}$$

Combining (16) and (17) we obtain the corresponding inverse transformations

$$x' = \gamma\left(x - Vt_E\right) \tag{18}$$

$$t'_R = \gamma\left\{\left[1 + \frac{V}{c}(1-n')\right]t_E - \left(1 - n' + \frac{V}{c}\right)\frac{x}{c}\right\}. \tag{19}$$

That approach is suggested by Yuan Zhong Zhang but not fulfilled. [3]

Equation (18) being synchrony parameter independent it holds in all "theories" that correspond to different values of the synchrony parameter n'.

3. Continuing with Reichenbach's philosophy

The equations we have derived above are presented by many authors. A review of them is presented by Yuan Zhong Zhang [3]. Edwards [4] derives them from (α) and (β_2) considering that in both inertial reference frames the synchronization of the clocks takes place with different synchrony parameters q and q' respectively. In the particular case when in I the clocks are standard synchronized (β_1) whereas in I' the clocks are nonstandard synchronized, the Edwards [4] transformations become using our notations

$$x' = \gamma\left((x - Vt_E\right) \tag{20}$$

$$t'_E = \gamma\left[\left(1 + \frac{V}{c}q'\right)t_E - \left(\frac{V}{c} + q'\right)\frac{x}{c}\right] \tag{21}$$

resulting that following Einstein's philosophy , we can consider that Edwards equations are the result an one way clock synchronization in I', using a synchronizing signal that propagates with speed (1-q') i.e with speed

$$c'_+ = \frac{c}{n'} = \frac{c}{1-q'} \tag{22}$$

resulting that for n'=1 and q'=0 we recover the Lorentz-Einstein transformations. q' being negative, the synchronizing signal in I' is subluminal.

If Edward's synchrony parameter dependent transformation equations were derived following Einstein's philosophy we could consider that all the "theories" that result for different values of the synchrony parameter n' are the consequence of the same philosophy.

4. Tangherlini [5] Selleri [6] and Abreu and Guerra [7]. Absolute simultaneity

The transformation equations proposed by the authors mentioned above and probably by many others are characterized by the fact that the transformation equation they propose, for the time coordinates, lead to absolute simultaneity ($\Delta t_E=0$ implies $\Delta t'_R$) We recover them from the synchrony parameter independent equation (20)

$$x' = \gamma\left(x - Vt_E\right) \tag{23}$$

and from (21) imposing the condition that they should be space coordinate independent which could be fulfilled imposing to n' and g' the values

$$1 - n' + \frac{V}{c} = 0 \tag{24}$$

$$q' + \frac{V}{c} = 0 \tag{25}$$

recovering the transformation equation proposed by those authors

$$t'_R = \gamma^{-1} t_E . \tag{26}$$

The peculiarities of the different ways in which equations (23) and (25) are derived could be found out from the original papers we quote.

5. An extension to two space dimensions

Introductory textbooks [8] perform the transition from one space dimensions to two ones by simply adding to the transformation equations derived so far the equation

$$y = y' \tag{27}$$

as a result of the fact that distances measured perpendicular to the direction of relative motion have the same magnitude in all inertial reference frames in relative motion. The proof of (27) does not involve light signal being merely a consequence of the relativity principle [9].

Considering the equations (23) and (27), both synchrony parameters independent we could state that all the physical quantities defined as a quotient of two lengths measured in the same inertial reference frame are synchrony parameter independent as well. As a first example we could consider the aberration of light effect [10] We introduce polar coordinates (r,θ) in I and (r',θ') in I' for defining the location of a point where events take place (r,r' lengths of position vectors, θ,θ' polar angles made by the position vectors with the positive direction of the permanently overlapped OX(O'X') axes. Combining (23) and (27) expressed as a function of polar angles we obtain that the lengths of the position vectors transform as

$$r' = \sqrt{x'^2 + y'^2} = r \frac{1 - \frac{V}{c}\cos\theta}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{28}$$

a n' independent transformation formula

What we compare in an aberration of light effect are the polar angles θ and θ' that define the directions along which the same signal propagates when detected from I and I' respectively. By definition

$$\cos\theta' = \frac{x'}{r'} = \frac{\cos\theta - \frac{V}{c}}{1 - \frac{V}{c}\cos\theta} . \tag{29}$$

which leads directly to

$$\cos\theta = \frac{\cos\theta' + \frac{V}{c}}{1 + \frac{V}{c}\cos\theta'} . \tag{30}$$

enabling us to present (28) as

$$r' = r\frac{\sqrt{1-\frac{V^2}{c^2}}}{1-\frac{V}{c}\cos\theta'}. \tag{31}$$

Equations (28) and (31) are largely used in special relativity theory. If $r=ct_E$ describes the wave front of a spherical wave emitted from O at t=0 detected from I then (28) describes the wave front of the same wave detected from I'. [11] If $t_E=R/c$ represent the geometric locus of simultaneous events in I then (28) represent their geometric locus in I' [12].

We underline that the transformation equations derived above have the same shape in all "theories" that correspond to different values of the synchrony parameter n'.

6. Kinematics in different theories corresponding to different values of the synchrony parameter n'

The definition of the speed of a given tardyon involves time the transformation of which involves the synchrony parameter n' and so does speed. By definition $u_{E,x}=\frac{x}{t_E}$ and $u'_{R,x}=\frac{x'}{t'_R}$ represent the speeds of the same tardyon measured in I using standard synchronized clocks whereas in I' using nonstandard synchronized ones we obtain that they transform (add) as

$$u'_{R,x}=\frac{u_{E,x}-V}{\left[1+\frac{V}{c}(1-n')\right]-\frac{u_{E,x}}{c}\left(1-n'+\frac{V}{c}\right)} \tag{32}$$

resulting that the origin O' ($u'_{R,x}=0$) of I' moves with speed V relative to I, the origin O of I (u_x=0 moving relative to I' with speed

$$V'_{R,x}=\frac{-V}{1+\frac{V}{c}(1-n')}. \tag{33}$$

Imposing the condition of absolute simultaneity (n'=1+V/c) the equations derived above become

$$u'_{R,x,n'=1+V/c}=\frac{u_{E.x}-V}{1-\frac{V^2}{c^2}} \tag{34}$$

and

$$V'_{R,x,n'=1+V/c}=-\frac{V}{1-\frac{V^2}{c^2}}. \tag{35}$$

The light signal that propagates with speed c relative to I propagates with speed $c'_{R,x}$ relative to I' with speed

$$c'_{R,x} = \frac{c}{1+\frac{V}{c}}. \tag{36}$$

Extending the problem to two space dimensions we derive the transformation equations for the OY(O'Y') components of the speed of the same tardyon starting with its definition in I'

$$u'_{R,y} = \frac{y'}{t'_R} = \frac{\sqrt{1-\frac{V^2}{c^2}}u_y}{\left[1+\frac{V}{c}(1-n')\right]-\frac{u_{E,x}}{c}\left(1-n'+\frac{V}{c}\right)}. \tag{37}$$

Introducing polar coordinates (32) and (37) become

$$u'_{R,x} = u_E \frac{\cos\theta - \frac{V}{u_E}}{\left[1+\frac{V}{c}(1-n')\right]-\frac{u_E}{c}\left(1-n'+\frac{V}{c}\right)\cos\theta} \tag{38}$$

$$u'_{R,y} = u_E \frac{\sqrt{1-\frac{V^2}{c^2}}\sin\theta}{\left[1+\frac{V}{c}(1-n')\right]-\frac{u_E}{c}\left(1-n'+\frac{V}{c}\right)\cos\theta} \tag{39}$$

the magnitudes of the speeds transforming as

$$u'_R = u_E \frac{\sqrt{\left(\cos\theta-\frac{V}{u}\right)^2+\left(1-\frac{V^2}{c^2}\right)\sin^2\theta}}{\left[1+\frac{V}{c}(1-n')-\frac{u_E}{c}\left(1-n'+\frac{V}{c}\right)\cos\theta\right]}. \tag{40}$$

If we replace the tardyon considered so far with a photon that propagates relative to I with speed c in all directions in space, equation (40) tells us that detected from I' its propagation is anisotropic being described by

$$c'_R = c\frac{1-\frac{V}{c}\cos\theta}{\left[1+\frac{V}{c}(1-n')-\left(1-n'+\frac{V}{c}\right)\right]} = \frac{1-\frac{V}{c}\frac{\cos\theta'+\frac{V}{c}}{1+\frac{V}{c}\cos\theta'}}{1+\frac{V}{c}(1-n')-\frac{\cos\theta'+\frac{V}{c}}{1+\frac{V}{c}\cos\theta'}\left(1+\frac{V}{c}-n'\right)}. \tag{41}$$

In the case of absolute simultaneity (41) becomes

$$c'_{R,n'=1+V/c} = \frac{c}{1+\frac{V}{c}\cos\theta'}. \tag{42}$$

resulting that in the direction of the O'Y' axis (θ'=π/2 and θ'=3π/2) anisotropy fades away.

7. The role of synchronization in relativistic dynamics

The problem is to find out synchrony parameter dependent transformation equations for the momentum and the energy of a tardyon. Observers from I knowing classical dynamics define the OX component of the momentum as

$$p_{E,x} = mu_{E,x} \tag{43}$$

whereas observers from I' will define it as

$$p'_{R,x} = m'_R u'_{R,x}. \tag{44}$$

Combining (43) and (44) we obtain

$$\frac{p'_{R,x}}{m'_R} = \frac{p_{E,x}}{m}\frac{1-\frac{V}{u_{E,x}}}{1+\frac{V}{c}(1-n')-\frac{u_{E,x}}{c}(1-n'+\frac{V}{c})}. \tag{45}$$

Equation (45) suggests considering that

$$p'_{R,x} = \Gamma\left(p_{E,x} - mV\right) \tag{46}$$

$$m'_R = \Gamma\left\{m\left[1+\frac{V}{c}(1-n')\right] - \frac{p_{E,x}}{c}\left(1-n'+\frac{V}{c}\right)\right\} \tag{47}$$

where Γ is an unknown function of the relative speed V but not of the physical quantities involved in the transformation process. We obtain its algebraic structure imposing the condition that for n'=1 (47) become the transformation equation proposed by Einstein's special relativity theory

$$m' = \gamma\left(m - \frac{V}{c^2}p_{E,x}\right) \tag{48}$$

where m and m' represent the relativistic masses of the same tardyon measured in the conditions that in I and I' the clocks are standard synchronized. The result is

$$\Gamma = \gamma = \left(1-\frac{V^2}{c^2}\right)^{-1/2} \tag{48}$$

and so we obtain the synchrony parameter independent transformation

$$p'_{R,x} = \frac{p_{E,x} - mV}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{49}$$

and the synchrony parameter dependent one

$$m'_R = \gamma \left\{ m \left[1 + \frac{V}{c}(1-n') \right] - \left(1 - n' + \frac{V}{c} \right) \frac{p_{E,x}}{c} \right\} \tag{50}$$

Defining $E_E=mc^2$ and $E'_R = m'_R c^2$ as being the energies of the same tardyon measured in I and in I' respectively (49) becomes

$$p'_{R,x} = \gamma \left(p_{E,x} - \frac{V}{c^2} E_E \right) \tag{51}$$

equation (50) becoming

$$E'_R = \gamma \left\{ \left[1 + \frac{V}{c}(1-n') E_E \right] - \left[1 - n' + \frac{V}{c} \right] c p_{E,x} \right\}. \tag{52}$$

In the case of absolute simultaneity (n'=1+V/c) (52) becomes

$$E'_R = \gamma^{-1} E_E .$$

Defining the OY and O'Y' components of the momentum as

$$p'_R = m'_R u'_{R,y} \tag{53}$$

$$p_{E,y} = m_E u_{E,y} \tag{54}$$

we obtain

$$p'_{R.y} = p_{E,y} \tag{55}$$

resulting that the (OY(O'Y') components of the momentum are not only synchrony parameter free but also invariant as well.

Generalizing the results obtained so far we can say that the equation that performs the transformation of the vector component of a four vector is synchrony parameter independent whereas the scalar component transforms according to a synchrony parameter dependent one.

8.Conclusions

The general synchrony parameter dependent Edwards transformations [4] could be derived following the principle of the invariance of the round trip of the speed of light but also using the ready derived Lorentz-Einstein transformations and performing the transformation of the space-time coordinates generated by the subluminal signal that performs the nonstandard synchronization when it arrives at the location of the clock to be synchronized. Different values of the synchrony parameter n' recover particular approaches to the problem. The problem is extended to two space dimensions and to the role played by clock synchronization in relativistic dynamics.

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